

Notes 1.2 – Functions & Inverses

Warmup – Solve for the variable

$$1. \quad \begin{array}{r} 17 = 5x + 2 \\ -2 \quad -2 \\ \hline 15 = 5x \end{array}$$

$$\boxed{x = 3}$$

$$2. \quad \begin{array}{r} 11 = \sqrt{2x+1} \\ |21 = 2x+1 \\ |20 = 2x \end{array}$$

$$\boxed{x = 10}$$

$$3. \quad \begin{array}{r} \sqrt{x^2+x-2} = 2 \\ x^2+x-2 = 4 \\ x^2+x-6 = 0 \\ (x+3)(x-2) = 0 \end{array}$$

$$\boxed{x = -3, 2}$$

$$4. \quad \begin{array}{r} -4 = \sqrt[3]{5x+1} \\ -64 = 5x+1 \\ -65 = 5x \end{array}$$

$$\boxed{x = -13}$$

$$5. \quad \begin{array}{r} 2x^2 - 5 = 3x^2 - 12x + 31 \\ -5 = x^2 - 12x + 31 \\ 0 = x^2 - 12x + 36 \\ 0 = (x-6)^2 \end{array}$$

$$\boxed{x = 6}$$

$$6. \quad \begin{array}{r} \sqrt[3]{352} = \sqrt[3]{7x^2+9} \\ 352 = 7x^2+9 \\ 343 = 7x^2 \\ 49 = x^2 \end{array}$$

$$\boxed{x = \pm 7}$$

$$7. \quad 9^x = 243$$

$$3^{2x} = 243$$

$$3^{2x} = 3^5$$

$$2x = 5$$

$$\boxed{x = \frac{5}{2}}$$

$$8. \quad 5^x = \frac{1}{125}$$

$$\boxed{x = -3}$$

Investigation

The faster you go the longer it takes to come to a complete stop. This is an important to remember when you are driving a car.

- a. List some factors that may affect how long it takes a car to come to a complete stop.

type of road
weather conditions
tire condition

brake condition
oil or gravel on road
condition of road
weight of car

Police departments and insurance companies have run many experiments to be able to mathematically model the relationship between the speed of a car and the stopping distance or braking distance (distance traveled after the brakes have been applied).

Let's use the Ferrari 550 Maranello, (an Italian sports car, vroom-vroom). Experiments have shown that on smooth, dry roads, the relationship between the braking distance (d) and speed (s) is given by $d(s) = .03s^2$. Speed is given in miles per hour (mph) and the distance is in feet.

- b. How many feet should you leave between you and the car in front of you if you are driving the Ferrari at 55 mph?

$$d(55) = .03(55)^2 \quad 90.75 \text{ feet of space}$$

- c. What distance should you keep between you and the car in front of you if you are driving 100 mph?

$$d(100) = .03(100)^2 \quad 300 \text{ feet of space}$$

- d. If an average car is about 16 feet long, about how many car lengths should you have between you and that car in front of you if you are driving 100 mph?

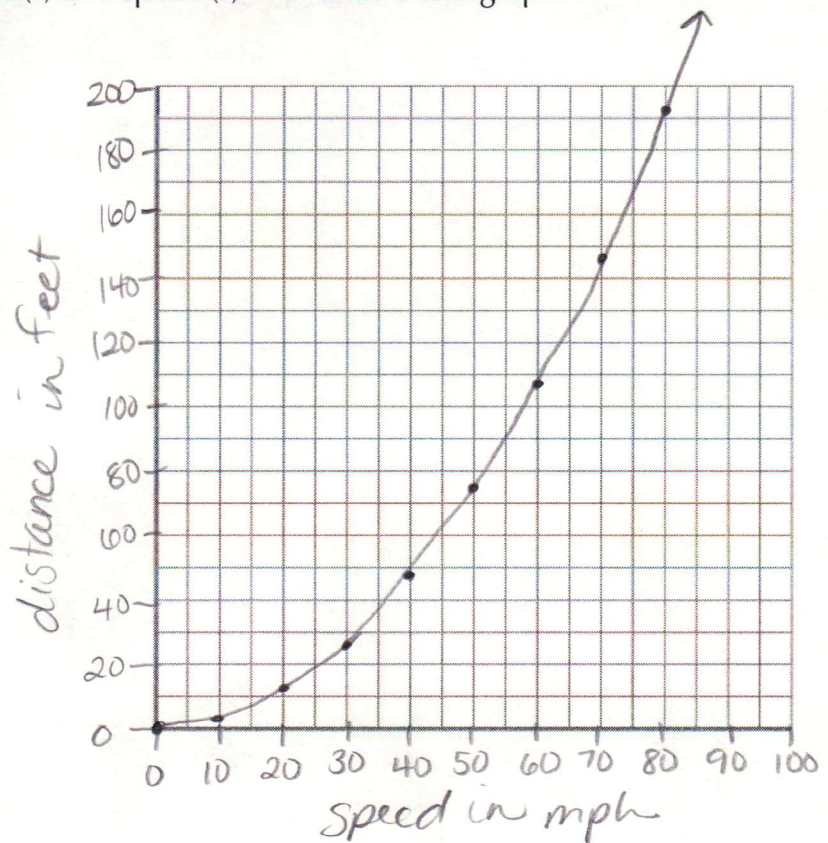
$$300 \div 16 \approx 18.75 \quad \text{about 19 car lengths between you}$$

- e. Some people believe that if a car is moving at a certain speed, then doubles the speed, that the stopping distance also doubles. Is this true? Explain why or why not.

No, using the above examples 100 is about double 55, however 55 mph requires about 91 feet and 100 mph requires 300 feet or more than triple the distance.

- f. Model the braking distance $d(s)$ and speed (s) with a table and graph.

s	$d(s)$
0	0
10	3
20	12
30	27
40	48
50	75
60	108
70	147



- g. The maximum speed of this Ferrari is 217 mph. Use this to describe the mathematical features of $d(s) = .03s^2$.

$$d(217) = .03(217)^2 \approx 1412.67$$

x & y intercepts are both zero

Domain $[0, 217]$

Range $[0, 1412.67]$

graph is increasing over the domain

quadratic function

- h. The driver of the Ferrari was driving along and had to brake suddenly to avoid a cat in the road. If the skid marks left by the car were 31 feet long, how fast was she going when she hit the brakes?

$$31 = .03s^2$$

$$1033.\bar{3} = s^2$$

$$s \approx 32.15$$

She was driving about 32 mph.

- i. If the cat was 15 feet away, what is the fastest she could be going and still avoid hitting the cat?

$$15 = .03s^2$$

$$500 = s^2$$

$$s \approx 22.36$$

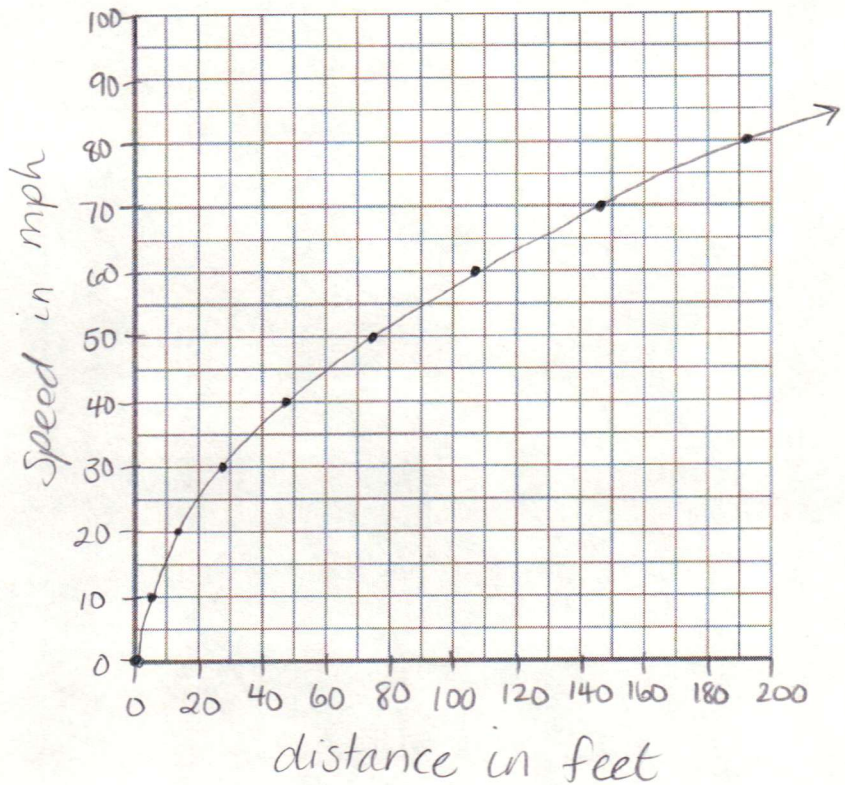
She could not go faster than about 22 mph.

- j. Write an equation $s(d)$ to model the speed of the car given the stopping distance (d).

$$s(d) = \sqrt{\frac{d}{.03}}$$

- k. Model the speed $s(d)$ for a given distance (d) with a table and graph.

d	$s(d)$
0	0
3	10
12	20
27	30
48	40
75	50
108	60
147	70
192	80



- l. Compare graphs of $d(s)$ and $s(d)$, what relationship do you see?

the inputs & outputs swap which makes the $x \leftrightarrow y$ axis swap - one is quadratic, the other is a square root graph

- m. Consider the function $d(s) = .03s^2$ over the domain of all real numbers, how does that change the shape of the graph?

the graph changes to a full parabola ↻

- n. How does changing the domain of $d(s)$ change the graph of the inverse of $d(s)$?

it would look like ↻

- o. Is the inverse of $d(s)$ a function? Explain.

No, if we do not limit the domain of $d(s)$ then $s(d)$ will not pass the vertical line test.